TR-PR-93-77

The nuclear component of the EMC effect beyond the Plane Wave Impulse Approximation

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(February 9, 2008)

Abstract

We study the response of a nucleus composed of nucleons and confined quarks. Assuming dynamics implied by a non-relativistic cluster model, we prove that the total response is a convolution of the responses of, respectively, a nucleus with interacting point-nucleons and an isolated nucleon with confined quarks. Defining as an intermediary structure functions in terms of the non-relativistic limit of light-cone variables, we subsequently conjecture a generalization to the relativistic regime. That result contains a nuclear part with full inter-nucleon dynamics and allows the study of approximations, as are the Plane Wave Impulse Approximation and a modification of it. The framework also permits a clean treatment of the response of a moving composite nucleon in the nucleus which may be off-shell.

I. INTRODUCTION

Consider the cross section for deep inelastic scattering of a weakly interacting probe by a nuclear target. Factoring out the elementary cross section, one is left with the nuclear structure function or linear response, which has been measured for various targets A. The outcome of the result per nucleon differs from structure function of a nucleon [1]. In spite of protracted efforts to describe those systematic deviations–the so-called EMC effect–there is as yet no consensus on a satisfactory explanation [2].

It is commonly accepted that at least the following two causes contribute to the EMC effect. 1) A redistribution of quarks ($'\alpha'$) in bound nucleons (N) which differ from the distribution in isolated ones [3,4]. 2) Nuclear effects due to binding, the interaction of nuclear fragments and the like [2,5,6]. It is the second alternative we shall pursue below.

The complexity of a relativistic treatment of the EMC effect provides the standard excuse for studying non-relativistic (NR) models. While being aware of the many features which have no NR analog, a solution of a model may furnish selective insight. As an example we mention a proof of asymptotic freedom of non-relativistically confined constituents [7].

Most treatments of the EMC effects have used the Plane Wave Impulse Approximation (PWIA) which leads to some form of a convolution for the structure function of the composite nucleus [8,9,10,11,12,13] and one is inclined to do the same in the NR approach. We shall formulate instead a cluster model with, as central property, separate quark and nucleonic degrees of freedom. By construction this assumption enables a clean analysis of the nuclear aspects of the EMC effect. No additional approximations are required to prove that the structure function of the fully interacting composite system is a convolution of structure functions for a nucleon and a nucleus composed of, respectively, quarks and point-nucleons. (Section II). Using the approach by Gersch-Rodriguez and Smith (GRS) [14], we consider the special case of high momentum transfer. It permits a separation of the structure function in an asymptotic part and additional Final State Interaction (FSI) contributions, ordered by increasing powers of the inverse momentum transfer. In Section III we consider the above

NR response in the PWIA for the nuclear part of the total response. In that treatment we shall be led to a precise formulation of the off-shell nature of the nucleonic component of the total response. As an alternative to the above we shall apply the West approximation to the nuclear part of the total response. In Section IV we first express structure function in terms of non-relativistic light-cone variables. Then independent of the underlying cluster model we conjecture the extension of the convolution formula into the relativistic regime.

Our final expression for the total nuclear structure function contains in principle the complete effect of nuclear dynamics and serves as a starting-point for the study of approximations. We thus discuss for the nuclear part a relativistic version of the PWIA and a modification, which includes some part of the FSI effects, neglected in the PWIA.

II. NON-RELATIVISTIC CLUSTER MODEL FOR THE RESPONSE OF A COMPOSITE NUCLEUS.

We consider a NR model for inclusive scattering of a weakly interacting probe from a target, transferring to the target momentum q and energy ν. Factoring out from the cross section for the process the same for the scattering of the probe from a constituent, one is left with, what is defined as the response or structure function of the target. For the latter we take a nucleus of A nucleons, each composed of 3 spinless $'$quarks$'$ of equal charge. With nucleon excitation energies much in excess of the same for nuclei, one is led to a cluster model with 3 quarks per nucleon with neglect of Pauli exchange between quarks in different nucleons. If the forces are confining, this is tantamount to a cluster model with 3-quark bags.

Quarks are thus assumed to move in a confining potential $V(r_{\ell,i})$, where $\boldsymbol{r}_{\ell,i}$ are coordinates of quark ℓ relative to $\boldsymbol{R}_i$, the center-of-mass coordinate of nucleon i with respect to the center-of-mass of the nucleus. For those quark coordinates $\sum_l \boldsymbol{r}_{l,i} = 0$. The nuclear potential we choose to be of the form $U = \sum_{j<i} U_{ij}(R_{ij})$, where $R_{ij} = |\boldsymbol{R}_i - \boldsymbol{R}_j|$. The above implies that the nuclear potential does not depend on the relative coordinates of the quarks $\boldsymbol{r}$' and

likewise, that the quark confining potential does not depend on $\boldsymbol{R}$'. Differently stated, we select Hamiltonians which act in separate spaces of quark and nucleon coordinates

$$H = \sum_{i=1}^{A} \left(\sum_{\ell=1}^{3} \left(-\frac{1}{2m} \nabla_{\ell,i}^2 + V(r_{\ell,i}) \right) - \frac{1}{2M} \nabla_i^2 + \sum_{j<i} U(R_{ij}) \right), \tag{1}$$

with m and M the quark and nucleon mass. As a consequence of the above assumption, states $|\Psi\rangle$ factorize in internal nucleon and nuclear states $|n\rangle, |N\rangle$ with respective energies e_n, E_N.

For large q the incoherent part dominates the structure function and its contribution per nucleon is

$$W_{\alpha/\mathrm{A}}(q,\nu) = 3 \sum_n \sum\!\!\!\!\!\!\int_N |\langle 0, 0| \exp\left[i\boldsymbol{q}(\boldsymbol{r}_{11} + \boldsymbol{R}_1)\right] |n, N\rangle|^2 \delta(\nu - E_{N0} - e_{n0} - q^2/2M_{\mathrm{A}}), \tag{2}$$

where $E_{N0} = E_N - E_0$; $e_{n0} = e_n - e_0$. The factor in front is the number of quarks per nucleon. Notice that the spectrum of confined quarks is always discrete, while nuclear states $|N\rangle$ may belong to either discrete or to continuum parts of a spectrum.

In order to evaluate the total structure function $W_{\alpha/\mathrm{A}}$ for the composite system, we consider first the structure function of an isolated nucleon. For use below, we also treat the general case of a nucleon in motion with initial momentum $\boldsymbol{P}$ and energy $\mathcal{E}$. The latter may be on shell, i.e. $\mathcal{E}^{on} = \mathcal{E}_{\boldsymbol{P}} = \boldsymbol{P}^2/2M$, but also arbitrary off-shell: $\mathcal{E} = \mathcal{E}^{off} \neq \boldsymbol{P}^2/2M$. The recoil momentum is $\boldsymbol{P}+\boldsymbol{q}$, and we shall need only the case of on-shell recoil energy, $\mathcal{E}_{\boldsymbol{P}+\boldsymbol{q}}$. The following expression

$$W_{\alpha/\mathrm{N}}(\boldsymbol{P}, \mathcal{E}; q, \nu) = 3 \sum_n |\langle 0| \exp\left(i\boldsymbol{q}\boldsymbol{r}_{11}\right) |n\rangle|^2 \, \delta\Big(\nu - e_{n0} + \mathcal{E} - \mathcal{E}_{\boldsymbol{P}+\boldsymbol{q}}\Big) \tag{3}$$

is thus the response for a moving, generally off-shell nucleon. As Eq. (3) shows, it is as follows related to the response of an on-shell nucleon at rest $W_{\alpha/\mathrm{N}}(q,\nu) \equiv W_{\alpha/\mathrm{N}}(0,0;q,\nu)$:

$$W_{\alpha/\mathrm{N}}(\boldsymbol{P}, \mathcal{E}; q, \nu) = W_{\alpha/\mathrm{N}}(q, \nu + \mathcal{E} - \mathcal{E}_{\boldsymbol{P}+\boldsymbol{q}} + \mathcal{E}_{\boldsymbol{q}}) \tag{4}$$

Next we need the response per nucleon of a nucleus of A point-like nucleons (in short the nuclear response)

$$W_{\mathrm{N/A}}(q,\nu) = \sum_N\!\!\!\!\!\!\int \; |\langle 0| \exp{(i\boldsymbol{q}\boldsymbol{R}_1)}\, |N\rangle|^2 \, \delta(\nu - E_{N0} - q^2/2M_{\mathrm{A}}) \tag{5}$$

From Eqs. (2), (3) and (5) one easily finds for the response of the system under consideration in terms of the same for a free nucleon and a nucleus of point nucleons

$$W_{\alpha/\mathrm{A}}(q,\nu) = \int_{q^2/2M_A}^{\nu} W_{\mathrm{N/A}}(q,\nu') W_{\alpha/\mathrm{N}}(q,\nu - \nu' + q^2/2M) d\nu' \tag{6}$$

Eq. (6) is an exact expression for the desired response in the cluster model and is a convolution of $W_{\alpha/\mathrm{N}}$ and $W_{\mathrm{N/A}}$: Nothing but the assumption on separability of spaces went in the derivation of the convolution (6). Notice that the latter may also be written as

$$W_{\alpha/\mathrm{A}}(q,\nu) = 3 \sum_n |\langle 0| \exp{(i\boldsymbol{q}\boldsymbol{r}_{11}|\, n\rangle|^2 \, W_{\mathrm{N/A}}(q, \nu - e_{n0} - q^2/2M) \tag{7}$$

Since the energy argument in $W_{\mathrm{N/A}}$ above is the same as in the expression (3) for the structure function of a free on-shell nucleon at rest ($\boldsymbol{P} = 0$), Eq. (7) shows that embedding of a nucleon in a nucleus broadens the δ-function peaks in (3), corresponding to excitations of the nucleon.

In the framework of our cluster model, which assumes separability of nucleon and quark interactions, the response $W_{\mathrm{N/A}}(q,\nu)$ is an observable quantity. By definition it is that part of the total response where nucleons are detected without associated particle production, i.e. in their ground state n=0. Thus

$$W_{\mathrm{N/A}}(q,\nu) \rightarrow \frac{\tilde{W}_{\alpha/\mathrm{A}}(q,\nu)}{\mathcal{F}_{\mathrm{N}}(q)}, \tag{8}$$

where $\mathcal{F}_{\mathrm{N}}(q) = 3\langle 0| \exp{(i\boldsymbol{q}\boldsymbol{r})}\, |0\rangle$ is the elastic form factor of the nucleon and $\tilde{W}_{\alpha/\mathrm{A}}(q,\nu)$ is the response spelled out above. In principle $\tilde{W}_{\alpha/\mathrm{A}}(q,\nu)$ can be extracted from A(e,e')X, A$(e,e'$N$)$X A$(e,e'$NN$)$X , . . . reactions in regions where in final states X particle production is kinematically forbidden. $\tilde{W}_{\alpha/\mathrm{A}}$ is the corresponding response integrated over all nucleon momenta.

We now address the high-q behavior of responses and shall use the Gersch-Rodriguez-Smith (GRS) representation for the response [14]. There one replaces the energy transfer parameter ν by the GRS-West variable [14,15]

$$y = -\frac{q}{2} + \frac{m\nu}{q} \tag{9}$$

and defines the reduced response

$$\phi(q,y) = (q/m)W(q,\nu) \tag{10}$$

In both (9) and (10), m is always the mass of the struck particle which absorbs the transferred (q,ν). As above we consider only the dominant incoherent part of the NR reduced response. As has been shown by GRS [14] the reduced response permits the expansion

$$\phi(q,y) = F_0(y) + (m/q)F_1(y) + (m/q)^2F_2(y) + \cdots, \tag{11}$$

where the coefficients F_i may be calculated as function of the interaction V between the constituents. In particular one has for the asymptotic limit

$$F_0(y) = \lim_{q\to\infty}\phi(q,y) = (q/m)\int n(p)\delta(\nu + \epsilon_{\boldsymbol{p}} - \epsilon_{\boldsymbol{p}+\boldsymbol{q}})d\boldsymbol{p} = \int n(p)\delta(y - p_z)d\boldsymbol{p}, \tag{12}$$

with $n(p)$, the single constituent momentum distribution. Eq. (12) shows that the scaling variable y is the minimal value of the momentum of the struck constituent in the direction of $\boldsymbol{q}$, when the constituent is on its energy shell before, as well as after the absorption of the virtual photon.

Note that in the asymptotic limit (12) there is no trace of the interaction V in the single particle energies $\epsilon_{\boldsymbol{p}} = \boldsymbol{p}^2/2m$. This holds, whether V is regular or singular, corresponding to confining forces [7]. However, that interaction is implicit in the exact momentum distribution in (12).

It is now a simple matter to rewrite the convolution (6) in terms of reduced responses

$$\phi_{\alpha/\mathrm{N}}(q,y) = 3\sum_n \left|\langle 0|\exp\left(i\boldsymbol{q}\boldsymbol{r}'_{11}\right)|n\rangle\right|^2 \delta\left(y + \frac{M-m}{2M}q - \frac{me_{n0}}{q}\right) \tag{13}$$

and

$$\phi_{\mathrm{N/A}}(q,Y) = \sum\!\!\!\!\!\!\!\int_N \left|\langle 0|\exp\left(i\boldsymbol{q}\boldsymbol{R}'_1\right)|N\rangle\right|^2 \delta\left(Y + \frac{M_A-M}{2M_A}q - \frac{ME_{N0}}{q}\right) \tag{14}$$

Notice that the appropriate GRS-West scaling variable in Eq. (14) is

$$Y = -\frac{q}{2} + \frac{M\nu}{q} = \frac{M}{m}\left(y + \frac{M-m}{2M}q\right) \tag{15}$$

One readily checks

$$\phi_{\alpha/\mathrm{A}}(q,y) = \int_{Y_{min}}^{Y_{max}} \phi_{\mathrm{N/A}}(q,Y)\phi_{\alpha/\mathrm{N}}\left(q, y - \frac{m}{M}Y\right)dY, \tag{16}$$

where $Y_{min} = -q/2(1 - M/M_A)$ and $Y_{max} = (M/m)y + (q/2)(M/m - 1)$.

Next we consider for fixed y asymptotic limits like Eq. (12) for all responses involved in (16)

$$\lim_{q\to\infty} \phi_{\alpha/\mathrm{N}}(q,y) = \ (q/m)\int n(p')\delta(\nu + \epsilon_{\boldsymbol{p}'} - \epsilon_{\boldsymbol{p}' + \boldsymbol{q}})d\boldsymbol{p}' = \int n(p')\delta(y - p'_z)d\boldsymbol{p}' \tag{17a}$$

$$\lim_{q\to\infty} \phi_{\mathrm{N/A}}(q,Y) = \ (q/M)\int \mathcal{N}(P)\delta(\nu + \mathcal{E}_{\boldsymbol{P}} - \mathcal{E}_{\boldsymbol{P} + \boldsymbol{q}})d\boldsymbol{P} = \int \mathcal{N}(P)\delta(Y - P_z)d\boldsymbol{P}, \tag{17b}$$

where $n(p')$ and $\mathcal{N}(P)$ are the single quark, respectively the single nucleon momentum distributions. Applying Eqs. (17) to (16) one obtains

$$\lim_{q\to\infty} \phi_{\alpha/\mathrm{A}}(q,y) = \int\int \mathcal{N}(P)n(p')\delta\Big(y - p'_z - (m/M)P_z\Big)d\boldsymbol{P}d\boldsymbol{p}' = \int \tilde{n}(p)\delta(y - p_z)d\boldsymbol{p}, \tag{18}$$

with the momentum distribution

$$\tilde{n}(p) = \int \mathcal{N}(P)n\Big(\boldsymbol{p} - (m/M)\boldsymbol{P}\Big)d\boldsymbol{P}, \tag{19}$$

generated by convoluting the distributions $\mathcal{N}$ and n. With $\boldsymbol{P}$ and $\boldsymbol{p}$ the momentum of the nucleon and the struck quark, $\boldsymbol{p} - (m/M)\boldsymbol{P}$ is the relative momentum of that quark with respect to the center-of-mass of the nucleon. (See Fig. 1 for the kinematics of the three situations considered: Eqs. (17), and (18)). Eq. (18) manifests that also quarks, confined in nucleons and in turn bound in nuclei, are asymptotically free.

We note from (18) that the location of the Quasi-Elastic Peak (QEP) in the asymptotic response is the same as for a nucleus of point-nucleons, namely at $y = 0$. Eq. (19) shows that the composite nature of the target, affects only the width given by $n(p)$: A q-dependent shift in the position of the above $'$asymptotic$'$ QEP is generated by higher order FSI contributions in (11) and is therefore not manifestly related to nuclear binding.

III. THE RESPONSE OF A COMPOSITE TARGET IN THE PWIA.

Virtually all treatments of the EMC effect are based on the PWIA [2,3,4,5,6,8,9,10,11,12,13] which we now briefly review. In that approximation for the nuclear part of the Hamiltonian (1) one neglects the interaction between a selected nucleon $'1'$ and the remaining (A-1)-particle core, i.e. $\sum_{j<i} U(R_{ij}) \to \sum_{2\leq j<i} U(R_{ij})$. Excited states in Eq. (5) then become

$$|N\rangle \to |N\rangle^{PWIA} = |(A-1)_{\lambda,-\boldsymbol{P}}; \boldsymbol{P}\rangle, \tag{20}$$

where $|(A-1)_{\lambda,-\boldsymbol{P}}\boldsymbol{P}\rangle$ denotes an excited state of a moving core with internal and kinetic energies $\mathcal{E}^C_\lambda, \mathcal{E}^C_{\boldsymbol{P}}$, and a free nucleon with momentum $\boldsymbol{P}$ and energy $\mathcal{E}_{\boldsymbol{P}}$. Substitution of (20) into (5) and the replacements $\sum\!\!\!\!\!\!\int_N \to \sum\!\!\!\!\!\!\int_\lambda \int d\boldsymbol{P} \to \int dE d\boldsymbol{P}$ lead to

$$\begin{aligned} W^{PWIA}_{\mathrm{N/A}}(q,\nu) &= \sum\!\!\!\!\!\!\int_\lambda \int |\psi_\lambda(\boldsymbol{P})|^2 \delta(\nu - \Delta_\lambda - \mathcal{E}^C_{\boldsymbol{P}} - \mathcal{E}_{\boldsymbol{P}+\boldsymbol{q}}) d\boldsymbol{P} &\text{(21a)}\\ &= \int dE \int d\boldsymbol{P} S_{\mathrm{N/A}}(\boldsymbol{P},E) \delta(\nu - E - \mathcal{E}^C_{\boldsymbol{P}} - \mathcal{E}_{\boldsymbol{P}\,+\,\boldsymbol{q}}) &\text{(21b)}\\ &= \int d\boldsymbol{P} S_{\mathrm{N/A}}(\boldsymbol{P}, \nu - \mathcal{E}^C_{\boldsymbol{P}} - \mathcal{E}_{\boldsymbol{P}\,+\,\boldsymbol{q}}) &\text{(21c)} \end{aligned}$$

where $\psi_\lambda(\boldsymbol{P}) = \langle (A-1)_{\lambda,-\boldsymbol{P}}; \boldsymbol{P}|0\rangle$, is the overlap of the ground state of the target at rest and an excited core state λ with a free nucleon. The function

$$S_{\mathrm{N/A}}(\boldsymbol{P},E) = \sum\!\!\!\!\!\!\int_\lambda |\psi_\lambda(\boldsymbol{P})|^2 \delta(E - \Delta_\lambda) \tag{22}$$

above is the single-particle spectral function in terms of the separation energy Δ_λ of a nucleon, when removed from the target ground state, and leaving the core in an excited state λ. One notices that the integrand in (21c) is the structure function for the semi-inclusive A$(e,e'p)$X reaction in the PWIA, (Fig. 2), i.e. the spectral function in (22) where the energy argument is replaced by the missing energy $\nu - \mathcal{E}^C_{\boldsymbol{P}} - \mathcal{E}_{\boldsymbol{P}+\boldsymbol{q}}$, as required by energy conservation [16]. The integral over the nucleon momentum $\boldsymbol{P}$ leads to the structure function for the totally inclusive A(e,e')X process in that approximation, with no produced particles in final states X (cf. Eq. (8)).

A pre-requisite for the application of the PWIA to the nuclear response is an explicit Hamiltonian. However, in view of the singular, confining inter-quark forces, and in line with many other authors, we do not detail the $'$quark$'$ part of H, or any approximation to it. Subsequent application of the PWIA to the remaining part of H requires, strictly speaking, that part to be free of quark degrees of freedom, which almost guides one to the cluster Hamiltonian (1). When in the following we shall refer to PWIA, we have in mind application of that approximation exclusively to the nuclear component in a convolution, thus

$$W_{\alpha/\mathrm{A}}^{PWIA}(q,\nu) \equiv \int_{q^2/2M_A}^{\nu} W_{\mathrm{N/A}}^{PWIA}(q,\nu')W_{\alpha/\mathrm{N}}(q,\nu-\nu'+q^2/2M)d\nu' \tag{23}$$

Substitution of (21a) into Eq. (23) yields (cf. Fig. 3)

$$\begin{aligned} W_{\alpha/A}^{PWIA}(q,\nu) &= \sum\!\!\!\!\!\!\int_{\lambda} \int d\boldsymbol{P}|\psi_\lambda(\boldsymbol{P})|^2 W_{\alpha/\mathrm{N}}(q,\nu-\Delta_\lambda-\mathcal{E}_{\boldsymbol{P}}^C-\mathcal{E}_{\boldsymbol{P}+\boldsymbol{q}}+\mathcal{E}_{\boldsymbol{q}}) \\ &= \int_{E_m}^{E_M} dE \int d\boldsymbol{P} S_{N/A}(\boldsymbol{P},E) W_{\alpha/\mathrm{N}}(\boldsymbol{P},\mathcal{E}^{off}(\boldsymbol{P},E);q,\nu), \end{aligned} \tag{24}$$

with upper and lower limits E_m, E_M, resulting from the integrates over the δ-function in (21b). Comparison with Eqs. (4) shows that the structure function $W_{\alpha/N}$ relates to a nucleon target with initial momentum $\boldsymbol{P}$ and off-shell energy

$$\mathcal{E} = \mathcal{E}^{off}(\boldsymbol{P},E) = -E - \mathcal{E}_{\boldsymbol{P}}^C = -\Delta_\lambda - \mathcal{E}_{\boldsymbol{P}}^C \tag{25}$$

Indeed, the PWIA prescription, displayed in Fig. 2 assigns the nuclear target and the spectator core (but not the struck particle) to be on their respective energy shells: Energy conservation then prescribes the off-shell energy (25).

As in the previous Section, we replace the energy transfer ν by the scaling variable y, Eq. (9), and obtain for the nuclear reduced response (21b)

$$\phi_{\mathrm{N/A}}^{PWIA}(q,Y) = \int dE \int d\boldsymbol{P} S_{\mathrm{N/A}}(\boldsymbol{P},E)\delta\left(Y - P_z - \frac{M}{q}(E+\mathcal{E}_{\boldsymbol{P}}^C+\mathcal{E}_{\boldsymbol{P}})\right) \tag{26}$$

The total reduced response corresponding to (23) thus becomes

$$\phi_{\alpha/\mathrm{A}}^{PWIA}(q,y) = \int dY \phi_{\mathrm{N/A}}^{PWIA}(q,Y)\phi_{\alpha/\mathrm{N}}(q,y-\frac{m}{M}Y) \tag{27}$$

Substitution of (26) in (27) and subsequent integration over Y yields

$$\phi^{PWIA}_{\alpha/\mathrm{A}}(q,y)=\int_{E_m}^{E_M}dE\int d\boldsymbol{P}S_{\mathrm{N/A}}(\boldsymbol{P},E)\phi_{\alpha/\mathrm{N}}\Big(\boldsymbol{P},\mathcal{E}^{off}(\boldsymbol{P},E);q,y\Big) \tag{28}$$

$\phi_{\alpha/\mathrm{N}}$ above is the response of a moving nucleon with off-shell energy $\mathcal{E}^{off}$, Eq. (25). It can be related to the same at rest using the equivalent of Eq. (4) in y

$$\phi_{\alpha/\mathrm{N}}(\boldsymbol{P},\mathcal{E}^{off};q,y)=\phi_{\alpha/\mathrm{N}}\Big(q,y-\frac{m}{M}P_z+\frac{m}{q}(\mathcal{E}^{off}-\mathcal{E}_{\boldsymbol{P}})\Big) \tag{29a}$$

$$\phi_{\alpha/\mathrm{N}}(\boldsymbol{P},\mathcal{E}^{on};q,y)=\phi_{\alpha/\mathrm{N}}\left(q,y-\frac{m}{M}P_z\right), \tag{29b}$$

Along (28), an alternative expression may be derived from Eqs. (26) and (27), integrating now over E

$$\phi^{PWIA}_{\alpha/\mathrm{A}}(q,y)=\frac{q}{M}\int dY\int d\boldsymbol{P}S_{\mathrm{N/A}}\left(\boldsymbol{P},\frac{q}{M}(Y-P_z)-(\mathcal{E}^C_{\boldsymbol{P}}+\mathcal{E}_{\boldsymbol{P}})\right)\phi_{\alpha/\mathrm{N}}(q,y-\frac{m}{M}Y) \tag{30}$$

Compare now Eqs. (28) and (30). The former contains a $\boldsymbol{P}$-dependent response of an *off*-shell, moving nucleon. In contradistinction, $\phi_{\alpha/\mathrm{N}}$ in (30) is the $\boldsymbol{P}$-independent response of an *on*-shell nucleon, which enables formal integration over the momentum $\boldsymbol{P}$ in the nuclear response, Eq. (30). In spite of their apparent dissimilarity, Eqs. (28), (30) are identical expressions for the total response in the PWIA, obtained by performing the Y, respectively the E integrations in Eqs. (26), (27).

We conclude this section by mentioning the alternative West approximation for the reduced nuclear response [15], where out of the GRS series, Eq. (11), only the first term $\phi^W_{\mathrm{N/A}}(y)=F_0(y)=\lim_{q\to\infty}\phi_{\mathrm{N/A}}(q,y)$, (17b) is retained. It leads to

$$\phi^W_{\alpha/\mathrm{A}}(q,y)=\int dY\phi^W_{\mathrm{N/A}}(Y)\phi_{\alpha/\mathrm{N}}(q,y-\frac{m}{M}Y)=\int d\boldsymbol{P}\mathcal{N}(\boldsymbol{P})\phi_{\alpha/\mathrm{N}}(\boldsymbol{P},\mathcal{E}^{on};q,\nu), \tag{31}$$

i.e. the nucleon response averaged over the nuclear Fermi momentum-distribution. Notice that, contrary to the PWIA expression Eq. (28), the above approximation causes the energy of the struck nucleon to be shifted back to its on-shell value, as in Eq. (29b). For underlying NR kinematics

$$\lim_{q\to\infty}\int_{E_m}^{E_M}dES_{\mathrm{N/A}}(E,\boldsymbol{P})=\int_0^\infty dES_{\mathrm{N/A}}(E,\boldsymbol{P})=\mathcal{N}(\boldsymbol{P}),$$

witn N, the single nucleon momentum distribution. One checks from Eqs. (28), (31) that in that limit the PWIA and the West approximations for $\phi_{\rm N/A}$ coincide.

IV. RELATIVISTIC EXTENSION

In order to perform a transition of the above results to the relativistic case, we introduce light-cone variables as an intermediary. Thus for a quark ($'\alpha'$), a nucleon and the nuclear target at rest with 4-momentum p, P and $P_{\rm A}$ we define correspondingly light-cone momenta for each kind of particles, thus $p_\pm = p_0 \pm p_z$; $P_\pm = P_0 \pm P_z$; $P^A_\pm = M_A$. In addition we shall need light-cone fractions $p_-/P^{\rm A}_-$; $P_-/P^{\rm A}_-$ which we choose to relate to a nucleon, and not to the actual nuclear target at rest. Thus $x_{\alpha/{\rm A}} \equiv p_-/M$; $x_{\rm N/A} \equiv P_-/M$.

Consider now the above light-cone variables in the NR limit. In accordance with Eqs. (17), $p_z \to y$; $P_z \to Y$ and we thus have the following correspondences [17]

$$x_{\alpha/A} = \frac{p_0 - p_z}{M} \to \xi_{\alpha/A} = \frac{m-y}{M} \tag{32a}$$

$$x_{N/A} = \frac{P_0 - P_z}{M} \to \xi_{N/A} = \frac{M-Y}{M} \tag{32b}$$

Those define structure functions of on-shell targets at rest in terms of those NR momentum fractions

$$f^{nr}_{\alpha/{\rm N}}(q,\xi) \equiv M\phi_{\alpha/{\rm N}}(q, m - M\xi) \tag{33a}$$

$$f^{nr}_{\rm N/A}(q,\xi) \equiv M\phi_{\rm N/A}(q, M - M\xi) \tag{33b}$$

$$f^{nr}_{\alpha/{\rm A}}(q,\xi) \equiv M\phi_{\alpha/{\rm A}}(q, m - M\xi), \tag{33c}$$

Eq. (29a) enables the extension of the above, to structure functions of a moving off-shell nucleon

$$f^{nr}_{\alpha/{\rm N}}(\boldsymbol{P},\mathcal{E};q,\xi) \equiv M\phi_{\alpha/{\rm N}}\left(q, y - \frac{m}{M}P_z - \frac{m}{q}\Delta\mathcal{E}\right) \cong f^{nr}_{\alpha/{\rm N}}\left(q, \frac{M\xi}{M(1-\Delta\mathcal{E}/q) - P_z}\right), \tag{34}$$

where $\Delta\mathcal{E} = \mathcal{E}_{\boldsymbol{P}} - \mathcal{E}$ is the off-shell energy shift. In the last step in (34) we have used the approximation

$$y - \frac{m}{M}P_z - \frac{m}{q}\Delta\mathcal{E} \cong m - \frac{M(m-y)}{M(1-\Delta\mathcal{E}/q)) - P_z}$$

Just as Eqs. (6), (16) express the nuclear response as a convolution in ν and a NR scaling variable y, one obtains a third representation of the total response in terms of the variables ξ. Substitution of (32) into (16), the use of the definitions (33) and the approximation $m - y + (m/M)Y \cong M(\xi_{\alpha/\mathrm{A}}/\xi_{\mathrm{N/A}})$ leads to

$$f^{nr}_{\alpha/\mathrm{A}}(q,\xi_{\alpha/\mathrm{A}}) = \int d\xi_{\mathrm{N/A}} f^{nr}_{\mathrm{N/A}}\left(q,\xi_{\mathrm{N/A}}\right) f^{nr}_{\alpha/\mathrm{N}}\left(q,\xi_{\alpha/\mathrm{A}} - \frac{m}{M}\xi_{\mathrm{N/A}} + \frac{m}{M}\right) \tag{35a}$$

$$\cong \int d\xi_{\mathrm{N/A}} f^{nr}_{\mathrm{N/A}}\left(q,\xi_{\mathrm{N/A}}\right) f^{nr}_{\alpha/\mathrm{N}}\left(q,\frac{\xi_{\alpha/\mathrm{A}}}{\xi_{\mathrm{N/A}}}\right) \tag{35b}$$

A particular case is the asymptotic limits for the structure functions in Eq. (35)

$$f^{nr}_{\alpha/\mathrm{N}}(\xi) = \lim_{q\to\infty} f^{nr}_{\alpha/\mathrm{N}}(q,\xi) = M\int d\boldsymbol{p}_\perp n(\boldsymbol{p}_\perp, m - M\xi) \tag{36a}$$

$$f^{nr}_{\mathrm{N/A}}(\xi) = \lim_{q\to\infty} f^{nr}_{\mathrm{N/A}}(q,\xi) = M\int d\boldsymbol{P}_\perp \mathcal{N}(\boldsymbol{P}_\perp, M - M\xi), \tag{36b}$$

i.e. probabilities to find, respectively a quark inside the nucleon and a nucleon inside the nucleus, with light-cone momentum fraction ξ. Eq. (33c) then becomes

$$f^{nr}_{\alpha/\mathrm{A}}(\xi_{\alpha/\mathrm{A}}) = \lim_{q\to\infty} f^{nr}_{\alpha/\mathrm{A}}(q,\xi_{\alpha/\mathrm{A}}) = M\int d\boldsymbol{p}'_\perp \tilde{n}\left(\boldsymbol{p}'_\perp, m - M\xi_{\alpha/\mathrm{A}}\right), \tag{37}$$

with $\tilde{n}$ as in Eqs. (18), (19).

Eqs. (35) are a nearly direct consequence of a strictly NR cluster model, which is of course radically different from a realistic rendition of the actual relativistic dynamics for the constituents. Notwithstanding, the above result bears clear features of relativistic expressions for the total nuclear structure function which appeared in the literature. This observation invites to dissociate the outcome (35) from the underlying model and to attempt an extension into the relativistic domain.

We first return to the relativistic light-cone fractions in Eq. (32) and then conjecture the validity of Eqs. (35), if NR quantities are replaced by relativistic ones. Thus with $Q^2 = q^2 - \nu^2$ and

$$f^{nr}(q,\xi) \to f(Q^2,x) \tag{38}$$

Eq. (35b) becomes

$$f_{\alpha/\mathrm{A}}(Q^2, x_{\alpha/\mathrm{A}}) = \int_{x_{\alpha/\mathrm{A}}}^{M_\mathrm{A}/M} dx_{\mathrm{N/A}} f_{\mathrm{N/A}}(Q^2, x_{\mathrm{N/A}}) f_{\alpha/\mathrm{N}}\Big(Q^2, \frac{x_{\alpha/\mathrm{A}}}{x_{\mathrm{N/A}}}\Big) \tag{39}$$

The definitions (32) connect the light-cone momenta P_-, p_- to the fractions x, and those in turn to the Bjorken variables. For instance $x_{\mathrm{N/A}} = (P_0 - P_z)/M$, while in the high q-limit the corresponding Bjorken variable becomes $x^{Bj}_{\mathrm{N/A}} = Q^2/2M\nu \to (q-\nu)/M$. Energy conservation, and the fact that the knocked-out nucleon is on its mass shell (cf. Fig. 2), fix in the same limit $P_0 = [(\boldsymbol{P}+\boldsymbol{q})^2 + M^2]^{1/2} - \nu \to P_z + q - \nu$. The two quantities approach therefore the same limit in the large q, ν limit: we conjecture that (39) holds with x interpreted as x^{Bj} for any finite Q^2.

Eq. (39) is our main result. It gives the distribution function of a fully composite nucleus in terms of the same for point nucleons and for an isolated nucleon. The emphasis is on the distribution $f_{\mathrm{N/A}}$, which in principle contains the complete inter-nucleon dynamics.

At this point we focus on the difference in treatment of inter-quark and inter-nucleon forces. No mention has been made of the former and we deliberately avoid a difficult, realistic calculation of $f_{\alpha/\mathrm{N}}$, the hard portion in the total structure function. Instead one would like to take it from data. In principle no such problem exists for a calculation of $f_{\mathrm{N/A}}$, the soft nuclear part of the total structure function, and which may be described using conventional nuclear physics. We now proceed with a discussion of two approximations for the nuclear structure function and start with the PWIA.

Assume that, as above, the nuclear states can be described non-relativistically and the same for derived quantities ψ_λ or the spectral function $S_{\mathrm{N/A}}$, Eqs. (20)-(22). The sole relativistic aspect retained in this soft component is the kinematics of the knocked-out nucleon. We thus write for the structure function $W_{\mathrm{N/A}}(q,\nu)$ in the relativistic PWIA (cf. Fig. 2)

$$W^{PWIA}_{\mathrm{N/A}}(Q^2,\nu) = \sum\!\!\!\!\!\!\int_\lambda \int d^4P\, 2M|\psi_\lambda(\boldsymbol{P})|^2 \delta\left[(P_0+\nu)^2 - (\boldsymbol{P}+\boldsymbol{q})^2 - M^2\right]\delta(P_0 - P_0^{off}), \tag{40}$$

where

$$P_0^{off} = M_A - [(M^\lambda_{A-1})^2 + \boldsymbol{P}^2]^{1/2} - \nu \cong M - \Delta_\lambda - \mathcal{E}^C_{\boldsymbol{P}} \tag{41}$$

is the off-shell total energy of the struck nucleon. The corresponding structure function $f_{\rm N/A}$ is obtained from $W_{\rm N/A}$ by replacing ν by $Q^2/2Mx_{\rm N/A}$. Thus using Eqs. (10), (33), (35b) and (39) the total nuclear response in the PWIA (see Fig. 3) becomes

$$f_{\alpha/{\rm A}}^{PWIA}(Q^2,x_{\alpha/{\rm A}})=\int_{x_{\alpha/{\rm A}}}^{M_A/M}dx_{\rm N/A}f_{\rm N/A}^{PWIA}(Q^2,x_{\rm N/A})f_{\alpha/{\rm N}}\Big(Q^2,\frac{x_{\alpha/{\rm A}}}{x_{\rm N/A}}\Big) \tag{42a}$$

$$f_{\rm N/A}^{PWIA}(Q^2,x_{\rm N/A})=\nu W_{\rm N/A}^{PWIA}(Q^2,Q^2/2Mx_{\rm N/A}), \tag{42b}$$

Consider now Eqs. (42) in the large-q limit $q^2\gg M^2+\boldsymbol{P}^2$. It permits to approximate to write in (40) $[(\boldsymbol{P}+\boldsymbol{q})^2+M^2]^{1/2}\to q+P_z$ while in the large-q limit $x_{\rm N/A}\to(q-\nu)/M$. As a result Eq. (42b) becomes

$$f_{\rm N/A}^{PWIA}(Q^2,x_{\rm N/A})\to\sum\!\!\!\!\!\!\int_\lambda\int d\boldsymbol{P}|\psi_\lambda(\boldsymbol{P})|^2\delta\Big(x_{\rm N/A}-\frac{P_0^{off}-P_z}{M}\Big) \tag{43}$$

Using the definition (22) of the single nucleon spectral function, substitution of (43) into (42a) gives on the one hand, after integration over $x_{\rm N/A}$ (cf. [18])

$$f_{\alpha/{\rm A}}^{PWIA}(Q^2,x_{\alpha/{\rm A}})=\int_{E_m}^{E_M}dE\int d\boldsymbol{P}S_{\rm N/A}(\boldsymbol{P},E)f_{\alpha/{\rm N}}\left(\boldsymbol{P},P_0^{off}(\boldsymbol{P},E);Q^2,x_{\alpha/{\rm A}}\right), \tag{44}$$

where (cf. Eq. (34))

$$f_{\alpha/{\rm N}}\left(\boldsymbol{P},P_0^{off}(\boldsymbol{P},E);Q^2,x_{\alpha/{\rm A}}\right)=f_{\alpha/{\rm N}}\left(Q^2,\frac{Mx_{\alpha/{\rm A}}}{P_0^{off}-P_z}\right) \tag{45}$$

is the structure function for a moving nucleon with off-shell energy $P_0^{off}=M-E-\mathcal{E}_{\boldsymbol{P}}^C$. On the other hand, integration over E leads to a nucleon structure function with no explicit $\boldsymbol{P}$ dependence

$$f_{\alpha/{\rm A}}^{PWIA}(Q^2,x_{\alpha/{\rm A}})=\int_{x_{\alpha/{\rm A}}}^{M_A/M}dx_{\rm N/A}f_{\rm N/A}^{PWIA}(x_{\rm N/A})f_{\alpha/{\rm N}}\Big(Q^2,\frac{x_{\alpha/{\rm A}}}{x_{\rm N/A}}\Big) \tag{46a}$$

$$f_{\rm N/A}^{PWIA}(x_{\rm N/A})=\int d\boldsymbol{P}S_{\rm N/A}\left(\boldsymbol{P},M(1-x_{\rm N/A})-P_z-\mathcal{E}_{\boldsymbol{P}}^C)\right) \tag{46b}$$

The expressions (44) and (46a) appear to differ in the explicit $\boldsymbol{P}$-dependence of the off-shell, respectively $\boldsymbol{P}$-independence of the on-shell nucleon structure function $f_{\alpha/{\rm N}}$. Those are actually identical as are the NR expressions (28), (30).

The PWIA is only one particular approximation for the nuclear part of the total response and parallel with the above NR development, we now discuss a possible relativistic version of the alternative West approach (cf. Eq. (31)). In the NR case the GRS theory provides the series (11), and in particular the asymptotic limit $F_0(y)$ in terms of the single nucleon momentum distribution $\mathcal{N}(P)$ and a scaling variable y. The latter we recall results in the NR case if, before and after absorption of the virtual photon, the nucleon is on its (energy-)shell. Regarding a relativistic generalization of y, the use of relativistic kinematics under similar on-mass shell conditions does not produce an acceptable expression. One needs in fact some theoretical framework for a proper definition. An example is the Bethe-Salpeter equation, where to lowest order as in the relativistic PWIA, the spectator is on its mass-shell. If the above nucleon before and after absorption of the virtual photon, is in equal measure kept off its mass-shell, i.e. $[P_0^2 - \boldsymbol{P}^2] = [(P_0 + \nu)^2 - (\boldsymbol{P} + \boldsymbol{q})^2]$. the above suffices for the definition of a proper relativistic analog of y [19]. One obtains

$$W^W_{\rm N/A}(Q^2,\nu) = 2M \sum\!\!\!\!\!\!\int_{\lambda} d^4P \, |\psi_\lambda(\boldsymbol{P})|^2 \delta\left[P_0^2 - \boldsymbol{P}^2 - (P_0+\nu)^2 + (\boldsymbol{P}+\boldsymbol{q})^2\right] \delta(P_0 - P_0^{off}) \quad (47)$$

which leads to the total response

$$\begin{aligned} f^W_{\alpha/{\rm A}}(Q^2, x_{\alpha/{\rm A}}) &= \int_{x_{\alpha/{\rm A}}}^{M_A/M} dx_{\rm N/A} f^W_{\rm N/A}(Q^2, x_{\rm N/A}) f_{\alpha/{\rm N}}\left(Q^2, \frac{x_{\alpha/{\rm A}}}{x_{\rm N/A}}\right) \\ f^W_{\rm N/A}(Q^2, x_{\rm N/A}) &= \nu W^W_{\rm N/A}(Q^2,\nu) \\ &= \int d\boldsymbol{P} S_{\rm N/A}\left(\boldsymbol{P}, M(1-x_{\rm N/A}) - \left(1 + \frac{4M^2 x^2_{\rm N/A}}{Q^2}\right)^{1/2} P_z - \mathcal{E}^C_{\boldsymbol{P}}\right), \end{aligned} \quad (48)$$

with P_0^{off} again as in (41). The above approximation and the relativistic PWIA, Eqs. (46b) differ for finite Q^2, but tend to the result (46b) for $Q^2 \to \infty$, $x_{\alpha/{\rm A}}$ =const. That limit appears not directly related to a Fermi averaged nucleon distribution function, as is the NR analog (31). Other treatments of FSI in the nuclear response will be mentioned in the Discussion.

V. SUMMARY AND DISCUSSION

We have studied above inclusive scattering from a nucleus with nucleons composed of spinless quarks. We assumed a model with triplets of quarks per nucleon, which do not interact with quarks in other nucleons. Likewise, the interactions between nucleons is independent of the positions of the constituents. The underlying assumptions are those of a cluster model with separate quark and nucleon dynamics. If the quark interactions confine, the specific cluster model is one with exclusively 3-quark bags.

The above cluster model is naturally realized for non-relativistic dynamics and implies then separated spaces for nucleons and quarks. Without the need to specify the involved interactions, the model suffices for a proof that the nuclear structure function for the composite system is a convolution of those functions for a nucleon and a nucleus composed of, respectively, quarks and fully interacting point-nucleons. The result generalizes the same used in the PWIA and is valid for any nucleon-nucleon (or nucleon-core) interaction beyond the PWIA. As such the model is an ideal tool to study nuclear facets of the EMC effect, in spite of the impossibility to produce excited states of the nucleons by the purely nuclear part of the cluster Hamiltonian.

Dependent on the choice of the external variable in addition to the momentum transfer q (e.g. the energy transfer ν or a scaling variable y) appropriate expressions can be given for the participating structure functions. We emphasized in particular the NR limits of light-cone variables. The form of the resulting total structure function has then been disconnected from the underlying cluster model and by proper change to the relativistic light-cone variables, assumed to hold in the relativistic regime. A novel feature is a proper inclusion in principle, of the full nucleon-nucleon interaction in the nuclear part of the Hamiltonian.

Starting from that result, we then studied two approximations. The first is the PWIA, which has been the standard tool for the investigation of the nuclear response, and where the interaction between a nucleon and the remaining core is neglected. We showed that in two equivalent expressions, one deals with the single nucleon spectral function as a remnant

of the nuclear response and the response of a moving nucleon. In those expressions, the nucleon is either on, or off its mass shell. No matter of principle is involved, since the two expressions can be transformed one into the other.

Our results are sufficiently general to encompass all previous formulations. In particular Eqs. (43), (44) and (46) are the precise formulations of the PWIA, without neglect of off-shell behavior in the response for the moving nucleon, where required. We refer in particular to a recent preprint by Melnitchouk *et al* [20] who study in detail deep inelastic scattering from an off-mass shell nucleon under general circumstances. The above authors assume that the nucleon response is not $\boldsymbol{P}$-dependent and perform the $\boldsymbol{P}$-integral on the remaining spectral function. As shown above, no such assumption is necessary and Eqs. (46) provide an alternative, where the P-integration has formally be performed.

Finally, as an alternative to the PWIA for the nuclear part of the total structure function, we considered a modification where the knocked-out particle is not on its mass-shell as in the PWIA, and has instead the same off-shell mass as before photon absorption. In this approximation, one retains at least some FSI between knocked-out nucleon and the core. The response in the PWIA and the above modification have the same asymptotic limit (43), but are quite different for finite Q^2 [19,21].

Till today only sporadic attempts have been made to go beyond the PWIA. Maybe the simplest reflection of FSI is the replacement $M \rightarrow M_{eff}$, and which has for instance been worked-out in [20]. A largely phenomenological treatment for finite A has been given by Frankfort et. al. [22]. Calculations for nuclear matter related to relatively large Q^2 $A(e,e')X$ reactions (and not the EMC effect) and which are based on nuclear dynamics are described in [21,23]. With emphasis in the past on the nuclear binding aspect of calculations of the EMC effect, it is clearly of interest to compare those with feasible determinations of FSI. Those calcuations are currently in progress and will be reported elsewhere.

VI. ACKNOWLEDGEMENT.

The authors would like to than Byron Jennings, Shim'on Levit and Roland Rosenfelder for discussions on the subject matter.

REFERENCES

[1] J.J. Aubert *at al*, Phys. Lett. **123**B, 285 (1983).

[2] See for instance the review by R.P. Bickerstaff and A.W. Thomas, J. Phys. **G15**, 1523 (1989).

[3] F.E. Close, R.G. Roberts and G.G. Ross, Phys. Lett. **142**B, 498 (1984); R.J. Jaffe, F.E. Close, R.G. Roberts and G.G. Ross, Phys. Lett. **134**B, 449 (1989).

[4] O. Nachtmann and H.J. Pirner, Zeitschr. f. Phys. C**21**, 277 (1984).

[5] S.V. Akulinichev, S.A. Kulagin and G.M. Vagradov, Phys. Lett. B**158**, 485 (1985); J. Phys. G**11**, L245 (1985); S.V. Akulinichev, S. Shlomo and G.M. Vagradov, Phys. Rev. Lett. **55** 2239 (1985); Phys. Rev. C**33**, 1551 (1986).

[6] G.V. Dunne and A.W. Thomas, Nucl. Phys. A**455**, 701 (1986); Phys. Rev. D**33**, 2061 (1986).

[7] S.A. Gurvitz and A.S. Rinat, Phys. Rev. C**47**, 290 (1993).

[8] W.B. Atwood and G.B. West, Phys. Rev. D**7**, 773 (1973).

[9] R.G. Roberts, *The Structure of the Proton*, Cambridge University Press, Cambridge, UK; 1990.

[10] R.J. Jaffe, Nucl. Phys. A**478**. 3c (1988).

[11] C.Ciofi degli Atti and S. Liuti, Phys. Lett. B225, 215(1989)

[12] Xiangdong Ji and B.W. Philipone, Phys. Rev. C42, R2279(1990)

[13] P.J. Mulders, A.W. Schreiber and H. Meyer, Nucl. Phys. A**549**, 498 (1992).

[14] H.A. Gersch, L.J. Rodriguez and Phil N. Smith, Phys. Rev. A**5**, 1547 (1972).

[15] G.B. West, Phys. Rep. C**18**, 264 (1975).

[16] See for instance S. Frullani and J. Mougey, Advances in Nuclear Phys. Vol. **14** (1984).

[17] The sign convention in Eqs. (32) is in agreement with $y < 0$, respectively $x > 1$ for elastic scattering.

[18] E. Pace and G. Salme, Phys. Lett. **110**B, 441 (1982).

[19] S.A. Gurvitz, Phys. Rev. C**42**, 2653 (1990).

[20] W. Melnitchouk, A.W. Schreiber and A.W. Thomas, preprint PSI-PR-93-13.

[21] A.S. Rinat and M.F. Taragin, Phys. Rev. C, in press.

[22] L.L. Frankfurt, M.I. Strikman, D.B. Day and M. Sargsyan, Rev. C**48**, 2451 (1993).

[23] , O. Benhar, A. Fabricioni, S. Fantoni, G.A. Miller, V.R. Pandharipande and I. Sick, Rev. C**44**, 2328 (1991).

Figure Captions.

Fig. 1a,b,c. Kinematics in the description of the asymptotic limits of structure functions with tragets at rest, corresponding to $W_{\alpha/\mathrm{N}}$, $W_{\mathrm{N/A}}$ and $W_{\alpha/\mathrm{A}}$ (Eqs. (17a), (17b) and (18)). Crosses on marks the particles, which are on shell in the GRS approach.

Fig. 2. Kinematics for nuclear structure function $W_{\mathrm{N/A}}$ in PWIA. Crosses mark on-shell particles. Figure illustrates non-relativistic and relativistic cases.

Fig. 3. Amplitude, underlying total nuclear structure function $W_{\alpha/\mathrm{A}}$ in PWIA. Photon vertex induces exact nucleon structure function $W_{\alpha/\mathrm{N}}$. Figure illustrates non-relativistic and relativistic cases.

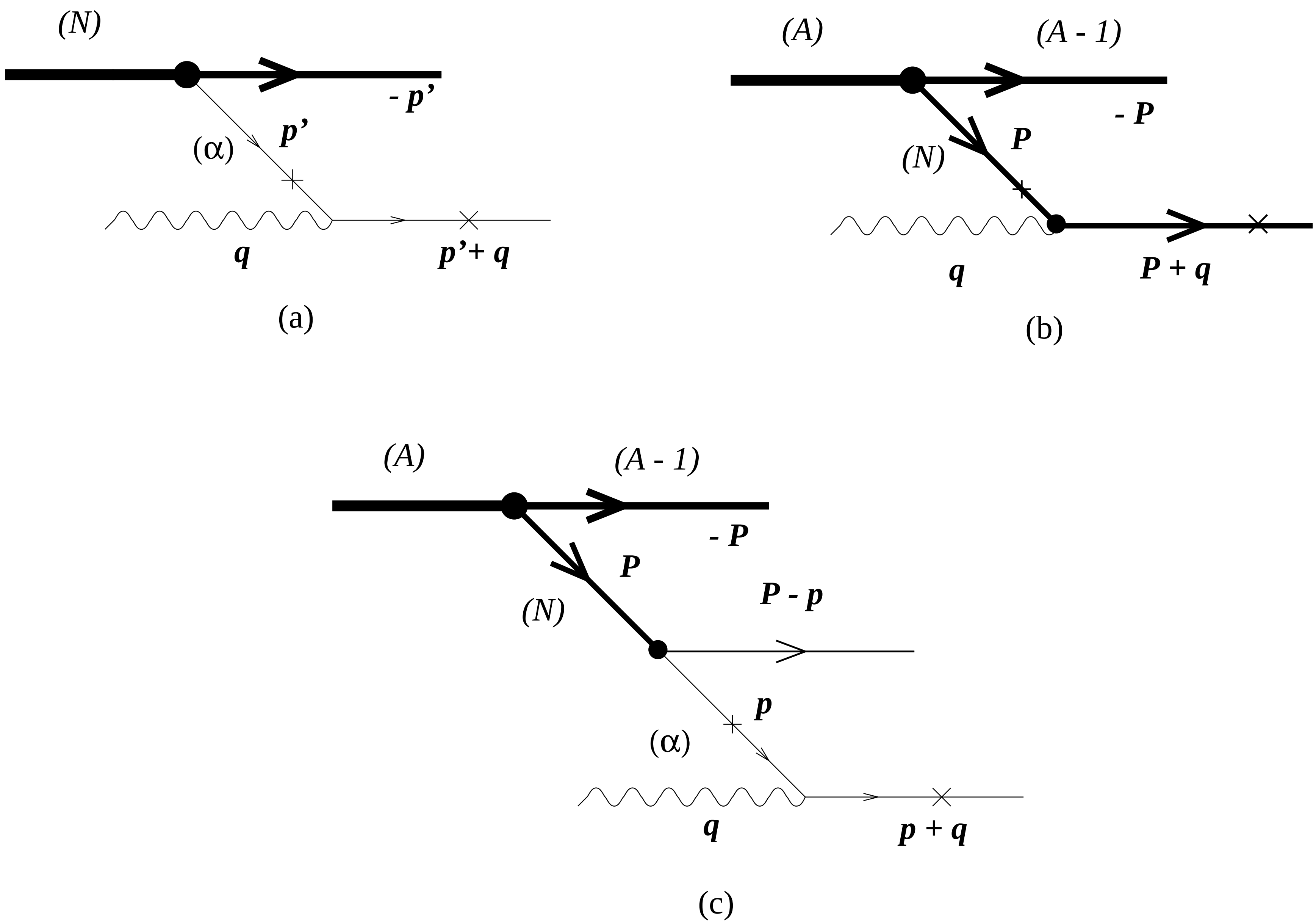

Fig. 1

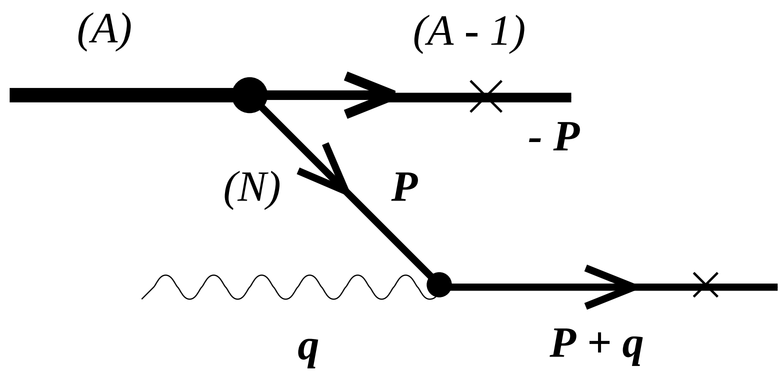

Fig. 2

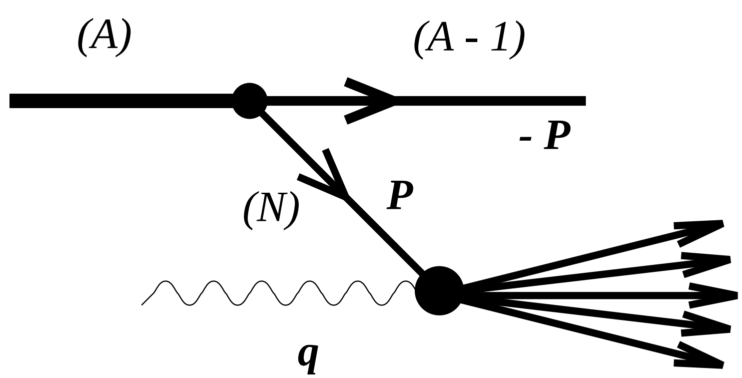

Fig. 3